STRATEGIES FOR SERVING PEAK ELEVATOR TRAFFIC

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1. INTRODUCTION

In designing a building (an office building in particular), the usual criteria for deciding how many elevators are needed is that one should be able to transport everyone in the building from some lobby floor to his destination (or vice-versa) within some period of time (20 min, for example) or to accommodate some percentage of the rush hour trips in some 5 min interval, Barney and dos Santos (1985), Strakosch (1985). How many people can be served in some time period, however, depends on how the elevators are employed. The maximum rate of serving passengers is achieved if each elevator carries a full load of passengers throughout its trip and makes a minimum number of stops per trip, i.e. each elevator goes from the lobby to only one floor with a full load and then returns to the lobby. It may serve a different floor on the next trip depending on what floors were served by the other elevators in the meantime.

This is not the strategy used for evaluating how many elevators are needed in a building and this is not the typical strategy used during the morning peak. It has, however, been used in some buildings which were, for some reason, designed with less than the customary number of elevators and which, therefore, had severe congestion problems in the morning peak. This is certainly not the design strategy recommended by elevator manufacturers who are in the business of selling elevators.

Another criteria used in elevator design is that a passenger’s waiting time (or average waiting time) should not exceed some specified value (1 minute, for example). If one decreases the number of floors served in each elevator trip, one will increase the time interval between trips to any given floor and, therefore, also increase the average waiting time (if the capacity of the elevator is sufficient to accommodate all waiting passengers on each trip going to the floors served in that trip). This may be used as an argument against having an elevator serve only one floor per trip since this may lead to waiting times exceeding the arbitrary specified allowed maximum.

Minimizing waiting time is not a reasonable objective for elevator strategy. Time spent riding a crowded elevator is at least as onerous as time spent waiting in a lobby. One might choose instead to minimize some weighted sum of waiting time plus riding time. If a passenger’s objective is simply to reach his destination as soon as possible, he would give equal weight to waiting or riding time. To determine how many elevators are needed one might adopt some maximum acceptable value for this.

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This paper is concerned mostly with moderately tall buildings (maybe 5 to 20 floors) which would be served by several elevators. Traffic during the peak will be predominantly between a single lobby floor and multiple upper floors. It is assumed that if there are some occasional trips between other floors they must (during the rush hour) go from their origin floor to the lobby and then make a separate trip from the lobby to their destination (or one could assign one or more elevators to serve only interfloor traffic). In the morning peak, traffic will be predominantly up-traffic and in the evening down traffic but there may be an uneven mixture of both up and down traffic.

For a tall building each elevator will be scheduled to serve some zone. Clearly, if an elevator served all floors in a tall building, the riding time of passengers would be unacceptably long even if the elevator cabin could hold all the passengers. The building would, therefore, be divided into zones and a certain number of elevators will be assigned to each zone. One might assign two or more elevators to any zone but, as we shall see, the main issue is whether one should assign two elevators to a zone or only one.

If the building already has been zoned so that only one or two elevators serve each zone, the number of floors in the zone would not likely be more than about six. During the peak period with nearly a full load of passengers on the elevator, the elevator is likely to stop at nearly every floor it is scheduled to serve, at least in the direction of heavier traffic.

A zone will consist of a set of adjacent floors. For the up-peak an elevator will run express to the lowest floor of the zone, visit all floors in the zone and then return to the lobby possibly stopping at some floors in the zone on the way down. Once the building has been divided into zones, the service within each zone is independent of that in any other zone.

2. FORMULATION

If the elevator runs non-stop from floor 1 to n and back to floor 1, the round trip time could be approximated by

$$2a + 2b(n - 1)$$

in which $b$ is the travel time per floor, $2b$ the travel time per floor going up plus coming down; $a$ is the time lost in accelerating, decelerating, opening and closing doors at each of the floors 1 and n. The value of $b$ is typically in the range of 1 to 2 s for a fast or slow elevator and $a$ is typically in the range of 10 to 15 s.

It is assumed that the elevator also stops at $k - 1$ intermediate floors (floors $n - k + 1$ to $n - 1$) at least in the direction of heavy traffic and possibly some fraction of these floors going in the opposite direction. The extra time to make these stops can be approximated by

$$a^*(k - 1)$$

$a^*$ being the time lost for each intermediate floor. If the elevator stops only in one direction, the $a^*$ may be somewhat less than the $a$ because this elevator might not reach cruise speed between adjacent floors, but, if the elevator stops also at some fraction of the floors in the other direction, the $a^*$ could be larger than $a$ (but less than $2a$).

In evaluating, the round trip time one may wish to add some time for loading and unloading passengers particularly at the lobby floor where this time may exceed the minimum time for the doors to stay open. This time will depend on the number of passengers carried per trip which in turn depends on the arrival rate of passengers. This, however, is not an important issue in comparing strategies. For now we suppose that this extra time can either be neglected or absorbed into the value of $a$.

The waiting time of a passenger is interpreted as the time from arrival at the elevator until the time that the elevator door closes, the last instant at which an arriving passenger can still board the elevator. The riding time will start from the moment the doors close. Thus any time when a passenger is on the elevator before the door closes will be interpreted as part of the waiting time.

If successive headways $H_i$ between dispatches to some floor are unequal, and a passenger arrives at some random time within a sequence of $n$ headways, his average waiting time is
\[
\left( \frac{1}{n} \sum_{i=1}^{n} H_i^2 \right) / 2 \left( \frac{1}{n} \sum_{i=1}^{n} H_i \right)
\]

Newell (1982). If there were two elevators serving a zone, the mean headway, \((1/n)\sum_{i=1}^{n} H_i\), is half the mean round trip. Since the elevator is assumed to make a predictable number of stops in each round trip, the round trip time is not expected to vary significantly from one trip to the next, so it may be convenient to write the average wait as some factor \(\beta\) times one quarter of the (mean) round trip time with.

\[
\beta = \left( \frac{1}{n} \sum_{i=1}^{n} H_i^2 \right) / \left( \frac{1}{n} \sum_{i=1}^{n} H_i \right)^{2/3}
\]

If one could control the headways so that successive headways were nearly equal \(H_i = H\) (as would be the case with only one elevator per zone), then \(\beta = H^2/H^2 = 1\). Otherwise \(\beta > 1\). But with two elevators we expect the round trip time to be nearly the same for both elevators and equal to the sum of any two (unequal) successive headways. The worst case would be for the two elevators to travel together with half the headways, in effect, zero and the other half equal to the round trip time. This would give a value of \(\beta = 2\). If the two elevators ran independently with no control, it might be reasonable to assume that the headway is random with a uniform distribution between 0 and the round trip time. This would give a value of \(\beta = 4/3\).

For this strategy with two elevators, the average waiting time per passenger is, therefore

\[
W_I = \beta/4[2a + 2b(n-1) + a^*(k-1)]
\]

If there are equal numbers of passengers alighting (or boarding) at each of the \(k\) floors, the average riding time of a passenger will be

\[
R_I = a + b(n-1) + (k-1)(a^* - b)/2
\]

The total waiting plus riding time is

\[
W_I + R_I = (1 + \beta/2)[a + b(n-1)] + (k-1)[a^*\beta/4 + a^*/2 - b/2]
\]

Now suppose that \(k\) is an even number and one elevator is assigned to each of the \(k/2\) adjacent floors. The average round trip time of the two elevators is

\[
2a + 2b[(n-1) - k/4] + a^*(k/2 - 1)
\]

The round trip time is nearly deterministic and each passenger can use only one of the elevators, so the average waiting time is now approximately half the average round trip time

\[
W_{II} = 1/2[2a + 2b((n-1) - k/4) + a^*(k/2 - 1)]
\]

and the riding time is

\[
R_{II} = a + b[(n-1) - k/4] = (k/2 - 1)(a^* - b)/2.
\]

As would be expected, the waiting time for strategy I is usually less than II. The part of the waiting time contributed by the non-stop trip time is less for strategy I by nearly a factor of \(\beta/2\) (1/2 for \(\beta = 1\)), but the part due to the stop time \(a^*\) is nearly the same for the two strategies. The first strategy benefits because the average headway is reduced by nearly a factor of 2 (for \(\beta = 1\)) but the second strategy benefits because the elevator makes only half as many stops.

The riding time for strategy II is less than strategy I. The part due to the non-stop trip time is, of course, the same for both strategies but the second strategy has only about half as much riding time spent in stops.

Perhaps the most important consideration in comparing these two strategies is the (average) number of passengers carried per trip (which cannot exceed the cabin capacity). For either strategy
this number will be proportional to the arrival rate of passengers $\lambda$, but the ratio of the average number of passengers carried for the two strategies is independent of $\lambda$. The average number of passengers carried per trip by strategy I is $\lambda$ times the average headway or $2\lambda W_I$ for $\beta = 1$. If in strategy II the arrival rate for each elevator is $\lambda/2$, this average is $\lambda W_{II}$. The ratio of these is $2 W_I/W_{II}$.

Clearly strategy I must carry a higher average number of passengers per trip but it also has a higher variation in the number of passengers carried on successive trips if successive headways are unequal.

3. EXAMPLES

Since there are so many parameters in this formulation, the only way to compare these strategies is to insert some typical numbers and see what happens. For the lowest zone $k = n - 1$, assumed here to be an even number $2,4,6,...$, this zone would certainly be served by a low speed elevator. If $b = 2\ s$, $a = a' = 10\ s$ (for one-way traffic) and $\beta = 1$, the times obtained (in seconds) are shown in Table 1.

The case $n = 3$ is rather academic (two elevators to serve two upper floors) but even in this case the waiting plus riding time is (slightly) less if one assigns one elevator to each floor. As the $n$ (and $k$) increase so does the advantage of assigning the elevators to separate floors (relative to waiting plus riding time).

If a more realistic value of $\beta$ had been taken, say $\beta = 4/3$, the waiting time for strategy I would be increased by a factor of $\beta$ making even $W_{II} < W_I$ for $k \geq 4$. If the elevator must make stops at all floors in both directions the advantage of strategy II is even larger as illustrated by Table 2 for $\beta = 1$. For two way traffic strategy II has an extra benefit because each elevator makes only one stop at the top floor of its trip to serve both up and down trips.

Actually the difference between the waiting times or the waiting plus riding times of these two strategies is not enough to justify any clear preference, but this does at least counter any argument that one should use strategy I because it (usually) gives less average waiting time.

The main advantage of strategy II is that it has a smaller average number of passengers carried per trip by the factors shown in the last column of Tables 1 or 2. For a specified cabin size, this also represents the ratio of the maximum stationary arrival rates that can be served by the two strategies or, for a given rush hour demand rate, the ratio of the cross section of elevator shafts needed to accommodate the peak demand.

In proceeding to higher zones strategy I gains some advantage because of the longer express run. Suppose, for example, four elevators served $2k$ floors $k = 2,4,.. \ (n = 2k + 1)$ with the upper zone having $k$ floors. Compare the upper zone with two elevators or splitting the zone with each elevator serving $k/2$ floors. For the same values of the parameters as in Table 1, the roundtrip time is increased by $2bk = 4k\ s$ , the time to traverse the lower zone non-stop, and the riding time increases by $2k\ s$, but the waiting time for strategy I increases by only $k\ s$ whereas that of strategy II increases by $2k\ s$. The times for the second zone are shown in Table 3 for one way traffic.

Table 1. One way traffic $\beta = 1$, lowest zone

<table>
<thead>
<tr>
<th>$k$</th>
<th>$n$</th>
<th>$W_I$</th>
<th>$W_{II}$</th>
<th>$R_I$</th>
<th>$R_{II}$</th>
<th>$W_I + R_I$</th>
<th>$W_{II} + R_{II}$</th>
<th>Average load I</th>
<th>Average load II</th>
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<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>9.5</td>
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<td>18</td>
<td>13</td>
<td>27.5</td>
<td>26</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>16.5</td>
<td>21</td>
<td>30</td>
<td>20</td>
<td>46.5</td>
<td>41</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>23.5</td>
<td>29</td>
<td>42</td>
<td>27</td>
<td>65.5</td>
<td>56</td>
<td>1.61</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>30.5</td>
<td>37</td>
<td>54</td>
<td>34</td>
<td>84.5</td>
<td>71</td>
<td>1.63</td>
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</tbody>
</table>

Table 2. Two way traffic $\beta = 1$, lowest zone

<table>
<thead>
<tr>
<th>$k$</th>
<th>$n$</th>
<th>$W_I$</th>
<th>$W_{II}$</th>
<th>$R_I$</th>
<th>$R_{II}$</th>
<th>$W_I + R_I$</th>
<th>$W_{II} + R_{II}$</th>
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</tr>
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<tbody>
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<td>3</td>
<td>12</td>
<td>13</td>
<td>18</td>
<td>13</td>
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<tr>
<td>4</td>
<td>5</td>
<td>24</td>
<td>26</td>
<td>30</td>
<td>20</td>
<td>54</td>
<td>46</td>
<td>1.85</td>
<td></td>
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<tr>
<td>6</td>
<td>7</td>
<td>36</td>
<td>39</td>
<td>42</td>
<td>27</td>
<td>78</td>
<td>66</td>
<td>1.85</td>
<td></td>
</tr>
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<td>8</td>
<td>9</td>
<td>48</td>
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<td>54</td>
<td>34</td>
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<td>1.85</td>
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</tr>
</tbody>
</table>
Except for $k = 2$, the waiting plus riding time for strategy II is still slightly less than for strategy I but the differences are too small to be of any importance. The main advantage of strategy II is that the elevator carries fewer passengers.

If one had six elevators with two of the elevators serving floors $2k + 2$ to $3k + 1$ one can make a similar comparison between serving these floors with both elevators or splitting the zone with each elevator serving $k/2$ floors. Table 4 shows the corresponding numbers for this zone.

It now seems that strategy I has finally gained a slight advantage of 2.5 s in waiting plus riding time but this is for one way traffic with $\beta = 1$. But also the building is now sufficiently tall that one would likely use a faster elevator. Even for $k = 4$ the elevator is running non-stop for eight floors.

Based on the above illustrations it would seem that the waiting plus riding time is rather insensitive to zoning a building with two elevators per zone or one elevator per zone. To accommodate the rush hour, however, with traffic predominantly between a single lobby floor and upper floors, the latter strategy could serve a significantly higher demand or would require less elevator shaft for a given demand.

4. ONE FLOOR PER TRIP

For very heavy traffic the optimal strategy is to have an elevator serve only one floor on each trip, since this maximizes the number of passengers which can be carried per unit time. But even if the carrying capacity is not an issue, this strategy might be satisfactory relative to waiting plus riding time.

Suppose now that a single elevator is assigned to serve floors $n - k + 1$ to $n$ ($k$ floors) with equal number of passengers going to each floor. For one strategy, corresponding to strategy II of the previous section, the elevator serves every floor of its zone on each trip. If the traffic is one-way it stops only in one direction but for two way traffic it may need to stop at each floor going up and down. For the next strategy, III, the elevator makes a trip to floor $n - k + 1$, returns to the lobby and then makes a trip to floor $n - k + 2$, etc. It is not expected that there will be very many floors per elevator, maybe $k = 2, 3, \text{ or } 4$.

For the lowest zone $n = k + 1$, Table 5 shows the waiting and riding times for the two strategies for the same values of the parameters as in the previous tables. The $W_{II}$ is for one-way traffic, $W_{II}$ for two-way traffic. The riding time is the same for both one-way and two-way traffic. For strategy III both the waiting time and riding time are the same for both one-way and two-way traffic since the elevator serves both the up and down traffic with one stop.

For an elevator serving just floors 2 and 3, the waiting plus riding time would be slightly less (37 vs 39 s.) if the elevator serves both floors on each trip rather than alternate trips to the two floors, if the traffic is only up traffic. If there is also down traffic, however, strategy III is slightly better (42 vs 39 s.). If $k = 3$ strategy III would make three separate trips to each of the three upper floors. For one-way traffic, the waiting plus riding time is still only slightly larger for strategy III (50 vs 56 s.) and is still slightly better (60 vs 56 s.) for two-way traffic. The main advantage of strategy III,
Table 5. One floor per trip, lowest zone

<table>
<thead>
<tr>
<th>k</th>
<th>n</th>
<th>$W_{II}$</th>
<th>$W'_{II}$</th>
<th>$W_{III}$</th>
<th>$R_{II}$</th>
<th>$R_{III}$</th>
<th>$W_{II} + R_{II}$</th>
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<tbody>
<tr>
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<td>19</td>
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<td>1.86</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>33</td>
<td>38</td>
<td>60</td>
<td>30</td>
<td>15</td>
<td>63</td>
<td>78</td>
<td>75</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Table 6. One floor per trip, second zone

<table>
<thead>
<tr>
<th>k</th>
<th>n</th>
<th>$W_{II}$</th>
<th>$W'_{II}$</th>
<th>$W_{III}$</th>
<th>$R_{II}$</th>
<th>$R_{III}$</th>
<th>$W_{II} + R_{II}$</th>
<th>$W_{III} + R_{III}$</th>
<th>Average load I</th>
<th>Average load II</th>
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<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>23</td>
<td>28</td>
<td>34</td>
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<td>23</td>
<td>79</td>
<td>94</td>
<td>115</td>
<td>1.87</td>
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however, is that it has fewer average passengers per trip as represented in the last column of Table 5, and can serve a considerably higher arrival rate.

As one goes to higher zones in the building strategy III becomes less advantageous because of the longer time it takes to return to the lobby floor. Table 6 shows the numbers for a second zone. Now strategy III has lost its advantage in terms of waiting plus riding time by a slight amount for $k = 2, n = 5$ but by a substantial amount for $k = 4, n = 9$. The latter, however, is rather extreme. It would correspond to having only two elevators serving a nine story building with each elevator serving four floors (with a slow elevator).

5. INTERPRETATION AND CONCLUSIONS

The main conclusion here is that it is inefficient to install an elevator system designed to accommodate the peak demand with a strategy in which more than one elevator serves a zone and each elevator stops at every floor of this zone. If the peak traffic is predominantly from or to a lobby floor, some (most) elevators should be assigned to serve this traffic in a manner such that each elevator serves only a few floors on each trip, possibly only one, seldom more than two.

If it is required to have an elevator serve different floors on successive trips, or to switch to a different strategy in the off-peak, or to allow for a change in strategy when there is some failure or servicing of the elevator, it would be advantageous to have a variable message sign at the lobby floor for each elevator showing which floors will be served by that elevator on its next trip. At floors other than the lobby one would not need any special equipment other than the conventional call button and a light indicating which elevator will stop next.

If there is unequal traffic to various floors the elevator could be programmed to follow some flexible strategy. The elevator system cannot measure how many passengers are waiting to go to any floor (or at any floor to go to the lobby). If buttons are installed at the lobby for each upper floor, the system could know if there is no demand for some floor because no one pushes the button, but it could not distinguish between several passengers pushing a button or one person pushing it many times. The elevator system, however, can estimate from the weight of the elevator approximately how many passengers boarded the elevator on all previous trips and could, therefore, estimate automatically the time-dependent average demand for each floor from ‘historical’ data. It could then dispatch elevators more frequently to floors with high demand.

Certainly there are simple strategies to serve rush hour traffic more efficiently than is customary now.

REFERENCES